EFFECT OF SURFACE TENSION ON THE COLLAPSE OF A CAVITATION BUBBLE

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By means of numerical calculations on the BÉSM-6 computer, we study the effect of surface tension on the size and rate of collapse of a cavitation bubble for the entire region of surface tensions of real liquids $\sigma = (1\text{-}60)\cdot 10^{-2}$ N/m (at temperatures $\simeq$ 290 K). The effect of two parallel solid walls bounding the liquid is taken into account.

The surface tension of the liquid is one of the basic factors determining the rate and nature of the collapse of cavitation bubbles, and hence the operation of various cavitation processes such as the erosion of solid surfaces, sonoluminescence, induced chemical reactions, etc. [1-3].

We consider a cavitation bubble collapsing in a gap between two parallel solid walls and having a radius $R = R_{max}$ at the initial time $t = 0$. We assume that the liquid is incompressible and inviscid and the effect of gravity is neglected. The compression of the gas—vapor mixture as the bubble collapses is assumed to be adiabatic with a ratio of specific heats $\gamma = 4/3$. We wish to determine the instantaneous radius R and the rate of collapse v of the bubble as functions of the surface tension of the liquid for different values of the gap width δ and the time. This situation often occurs in practice for various ultrasonic processes involving the cavitation processing of solid surfaces [1].

The problem is solved using the mathematical model of [4-6], where in calculating the motion, the velocity potential φ of the liquid and the radius R of the collapsing bubble are written as series in Legendre polynomials and Laplace's equation for φ is solved taking into account the boundary conditions on the surfaces of the solid walls and on the surface of the cavitation bubble, the boundary condition far from the bubble, and the initial conditions.

The effect of surface tension comes in through the Cauchy—Lagrange integral, which serves as one of the boundary conditions for the velocity potential of the liquid on the surface of the bubble:

$$\partial\varphi/\partial t + (\partial\varphi/\partial r)^2/2 + (\partial\varphi/\partial\theta)^2/2r^2 = (P_0 - P_R)/\rho \text{ for } r = R, \tag{1}$$

where r and θ are spherical coordinates with the origin at the center of the bubble.

Assuming the pressure of the liquid and the pressure inside the bubble P_b are equal on the liquid—bubble boundary, we have [2, 3]

$$P_R = P_b - 2\sigma/R. \tag{2}$$

Substituting (2) into (1), we have

$$\partial\varphi/\partial t + (\partial\varphi/\partial r)^2/2 + (\partial\varphi/\partial\theta)^2/2r^2 = A(R) + B(R)\sigma \text{ for } r = R, \tag{3}$$

where $A(R) = P_0/\rho - \alpha P_0 R_{max}{}^4/\rho R^4$; $B(R) = 2/\rho R$.

The boundary condition (3) leads to surface-tension-dependent terms in the steady-state system of ordinary first-order differential equations for the dimensionless coefficients x_0 and $x_{n,2m}$ of the bubble radius and y_0, y_{2m} of the rate of collapse ($n = 1, 2, 3, 4$; $m = 0, 1$; $2m < n$). Because of its complexity, this system of equations is not written out here.

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TABLE I. Effect of Surface Tension on the Size and Rate of Collapse of a Cavitation Bubble

λ	τ	$\overline{R}_0/\overline{R}_{\overline{\sigma}}$	$\overline{v}_{\overline{\sigma}}/\overline{v}_0$
1/20	0,2	1,0019	1,1057
	0,4	1,0090	1,1189
1/10	0,2	1,0017	1,1055
	0,4	1,0081	1,1178
1/6	0,2	1,0015	1,1052
	0,4	1,0069	1,1168

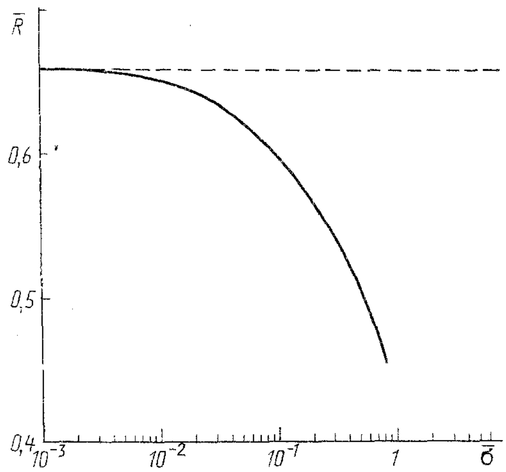

Fig. 1

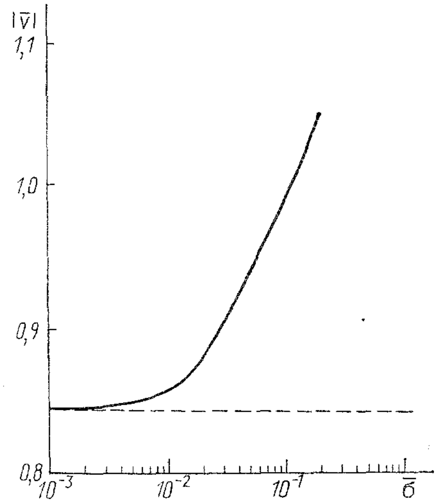

Fig. 2

Fig. 1. Dependence of the instantaneous dimensionless radius $\bar{R}$ of a collapsing cavitation bubble on the dimensionless surface tension $\bar{\sigma}$ for $\lambda = 1/10$ and a dimensionless time of $\tau = 0.8$. The dashed line corresponds to the solution for $\bar{\sigma} = 0$.

Fig. 2. Dependence of the absolute value of the rate of collapse of the cavitation bubble $|\bar{v}|$ on the dimensionless surface tension $\bar{\sigma}$ for $\lambda = 1/10$ and $\tau = 0.7$; the dashed line corresponds to the solution for $\bar{\sigma} = 0$.

The resulting system of equations was integrated numerically on the BÉSM-6 computer using the Merkon method with the integration step size determined dynamically. The relative accuracy is 10^{-7} for $\alpha = 0.02$, which corresponds to the gas content of real cavitation bubbles, [7], and for different values of the surface tension. Then the instantaneous radius and rate of capture of the cavitation bubble were determined from the equations

$$\overline{R} = R/R_{\max} = x_0 + \sum_{n=1}^{\infty} \lambda^n \Big(\sum_{m=0}^{k} x_{n,2m} P_{2m} \Big), \quad 2k < n, \tag{4}$$

$$\bar{v} = vL^{-1/2} = y_0 + \sum_{n=1}^{\infty} \lambda^n \Big(\sum_{m=0}^{k} y_{n,2m} P_{2m} \Big), \quad 2k < n. \tag{5}$$

Equation (3) is correct our spherical bubbles, i.e., for $\lambda \leq 1/10$ or for larger values of λ but for small values of the time, when the collapsing bubble is still spherical [4-6]. The basic results are shown in Figs. 1 and 2 and in Table I.

CALCULATED RESULTS

Figure 1 shows the typical dependence of the dimensionless instantaneous radius $\bar{R}$ of the bubble on the dimensionless surface tension $\bar{\sigma} = 2\sigma/\rho R_{max}L$. We see from Fig. 1 that the larger the surface tension, the smaller the size of the bubble for the same dimensionless time $\tau = tL^{\frac{1}{2}}/R_{max}$ and gap width δ, i.e., the surface-tension forces speed up the collapse process. When the surface tension σ increases by two orders of magnitude, from 10^{-3} to 10^{-1}, the bubble radius $\bar{R}$ decreases by only 10%. Hence the surface tension has a significant effect on the bubble size when

$$\bar{\sigma} \geqslant 10^{-1} \text{ or } \sigma \geqslant 0{,}05 R_{max} P_0. \tag{6}$$

Putting $P_0 = 10^5$ N/m² (atmospheric pressure), we obtain from (6)

$$\sigma \geqslant 5 \cdot 10^3 \, (\text{N/m}^{2}) \, R_{max} \, (\text{m}). \tag{7}$$

Hence the smaller the initial bubble size R_{max}, the smaller the value of σ for which surface tension is significant. This is because the pressure due to surface-tension forces P_σ is inversely proportional to the bubble size: $P_\sigma = 2\sigma/R$. For $R_{max} \simeq 10^{-4}$ m (a value typical for many ultrasonic technological processes based on cavitation [8]) we have

$$\sigma \geqslant 0{,}5 \text{N/m} . \tag{8}$$

For example, the coefficient of surface tension of water is equal to $7.6 \cdot 10^{-2}$ N/m and so surface tension can be neglected in calculating the process of collapse of cavitation bubbles in water.

For different gap widths and times, Table I gives the ratio of the instantaneous radius of a collapsing cavitation bubble $\bar{R}_0$ calculated for $\bar{\sigma} = 0$ to the radius $\bar{R}_{\bar{\sigma}}$ obtained assuming a surface tension of $\bar{\sigma} = 0.1$. The values of the ratio $\bar{v}_{\bar{\sigma}}/v_0$, defined similarly, are also given in Table I. We see that the effect of surface tension increases with increasing gap width (for a constant value of τ) and with time (for a constant value of λ). This is explained by the decrease in the instantaneous radius of the bubble [4-6], which leads to an increase in P_σ, and hence to an increase in the effect of surface tension on the size and rate of collapse.

Figure 2 can be used to estimate the effect of surface tension on the cavitation operation of different liquids used in ultrasonic technology, since the rate of collapse determines the destructive action of the bubbles. Knowing the surface tension of the liquid, one can determine from Fig. 2 how much surface tension speeds up the process of collapse compared to the "zero" level (the dashed line). On the other hand, if we know the necessary level of cavitation action on the solid boundary surfaces, as determined for a standard liquid with known physical properties (such as water), then Fig. 2 can be used to determine the optimum properties of the liquid by choosing the surface tension such that the required rate of collapse of cavitation bubbles is achieved (as in the standard liquid).

We note that for very narrow gaps between the solid boundary surface ($\lambda > 1/10$) the effect of the solid walls is so great that the bubble may deviate from a sphere during its collapse [4, 9-12]. In this case

$$P_\sigma = 2\sigma/R_c, \tag{9}$$

where R_c is the radius of curvature of the surface of the bubble at a given point on the surface. In a plane passing through the center of the bubble parallel to the solid walls, where the elongation of the bubble is a maximum, the radius of curvature of its surface is a maximum and therefore the pressure due to the surface-tension forces is a minimum (Eq. 9).

In this region the surface tension will have the smallest effect on the rate of collapse of the bubble and hence the deviation of the surface from a sphere will decrease. When there are dimples on the surface of the bubble, i.e., in the case when a radial microstructure exists in the liquid [4, 9], the surface-tension force changes sign in the dimples and points into the liquid. Hence surface tension slows the collapse process, whereas in the other regions of the surface of the bubble, surface tension will speed up the collapse. As a result, the deviation of the surface from a sphere decreases and therefore in liquids with large surface tension one expects a qualitative change in the nature of cavitation in a narrow gap: a transformation from a "jet" collapse to a spherically symmetric collapse, such that the results obtained here can be used.

CONCLUSIONS

1. By means of numerical calculations, we have obtained the dependence of the radius and rate of collapse of a cavitation bubble on surface tension, where the effect of parallel solid walls bounding the liquid is taken into account.

2. The effect of the surface tension increases as the gap width and time increase and is significant for $\bar{\sigma} \geq 0.1$, which corresponds to $\sigma \geq 0.5$ N/m in the case of ultrasonic processes of interest in technology.

3. The dependence of the rate of collapse of cavitation bubbles on the surface tension obtained here (Fig. 2) can be used in practice in the optimization of ultrasonic technological processes associated with cavitation processing of solid surface (purification, permeation, capillary control using ultrasound, and so on)

a) to estimate the cavitation operation of liquids as a function of the surface tension;

b) to obtain optimum properties of liquids by choosing the surface tension such that a required rate of collapse of cavitation bubbles is achieved.

NOTATION

Here σ is the coefficient of surface tension of the liquid; t, time; R, instantaneous radius of the cavitation bubble; R_{max}, bubble radius at the initial time $t = 0$; γ, ratio of specific heats of the gas—vapor mixture inside the collapsing bubble; v, rate of collapse of the bubble; δ, gap width; φ, velocity potential of the liquid; r, θ, spherical coordinates; P_0, P_R, pressure of the liquid far from the bubble and on the bubble—liquid boundary; P_b, pressure inside the bubble; ρ, density of the liquid; $A(R) = P_0/\rho - \alpha P_0 R_{max}{}^4/\rho R^4$; $\alpha = P_n/P_0$, gas-content parameter of the cavitation bubble; P_n, pressure of the gas—vapor mixture inside the bubble at $t = 0$; $B(R) = 2/\rho R$; x_0, $x_{n,2m}$, y_0, $y_{n,2m}$ ($n = 1, 2, 3, 4$; $m = 0, 1$; $2m < n$), dimensionless coefficients for the radius and rate of collapse of the bubble; $\bar{R} = R/R_{max}$; $\lambda = R_{max}/\delta$; P_{2m}, Legendre polynomial of order $2m$; $\bar{v} = vL^{-\frac{1}{2}}$; $L = P_0/\rho$; k, n, summation indices; $\bar{\sigma} = 2\sigma/\rho R_{max} L$; $\tau = tL^{\frac{1}{2}}/R_{max}$; $P_\sigma = 2\sigma/R$, pressure due to surface-tension forces; $\bar{R}_0$, $\bar{v}_0$, the values of $\bar{R}$ and $\bar{v}$ when $\bar{\sigma} = 0$; $\bar{R}_{\bar{\sigma}}$, $\bar{v}_{\bar{\sigma}}$, values of $\bar{R}$ and $\bar{v}$ for $\bar{\sigma} = 0.1$; R_c radius of curvature of the surface of the bubble.

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